

# MMME2044 Introduction to Brakes and Clutches

- Brakes and Clutches transfer the momentum/energy/force between two components capable of relative motion
- Normally we are considering rotating systems
- The mechanism for this transfer is friction
- Contacting surfaces generate heat due to friction and must meet certain requirements to perform effectively
- For brakes one component of the system is fixed

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# Key principles – Properties of frictional materials

Frictional material for brakes and clutches:

- High coefficient of friction
- Unaffected by moisture or heat
- Good heat conductivity
- High contact pressures ( $p_{\max}$ )
- High resistance to wear and scoring

Lubrication can be used to assist with heat transfer

Moulded friction material (lining):

aramid fibres + metallic particles + phenolic resin

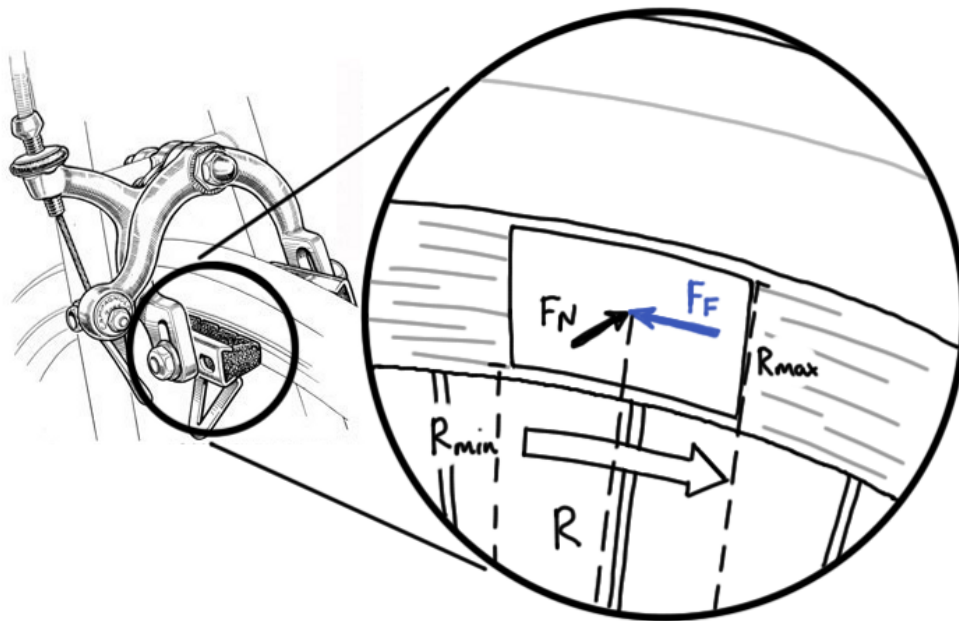
Car Brake Lining



# Rim brake

The common principle for all arrangements of brakes and clutches is that they rely on friction to transfer momentum/energy/force.

Perhaps the simplest example is the bicycle rim brake



Force is applied to the brake pad by the calliper

The force is normal to the surface

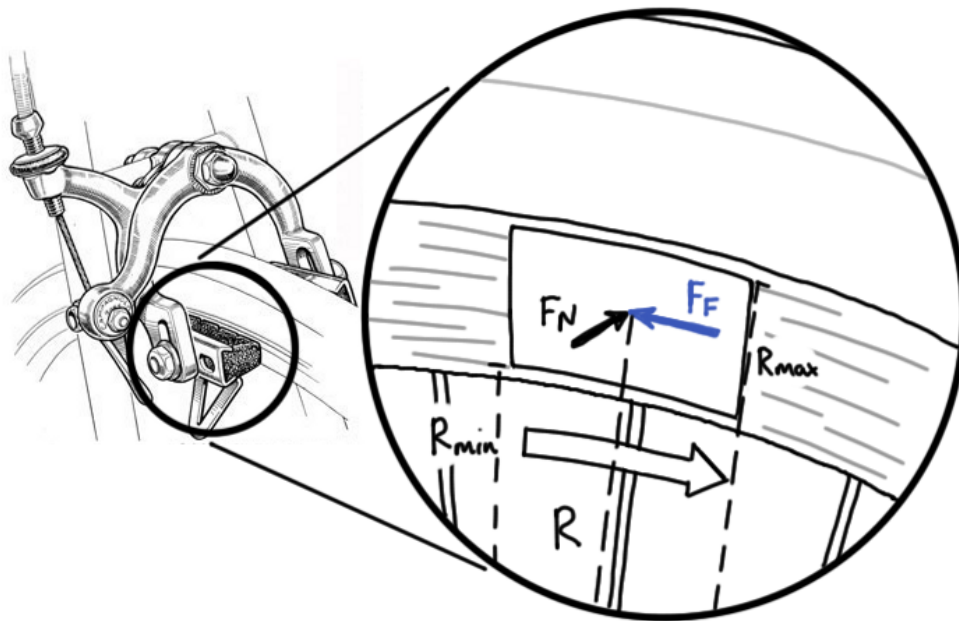
The amount of force is limited by the maximum rated pressure of the brake pad material,  $P_{max}$

$$P = F_N / \text{Area}$$

# Rim brake

The common principle for all arrangements of brakes and clutches is that they rely on friction to transfer momentum/energy/force.

Perhaps the simplest example is the bicycle rim brake



Normal force is distributed evenly

Friction force is therefore distributed evenly

Radius to the centre is large

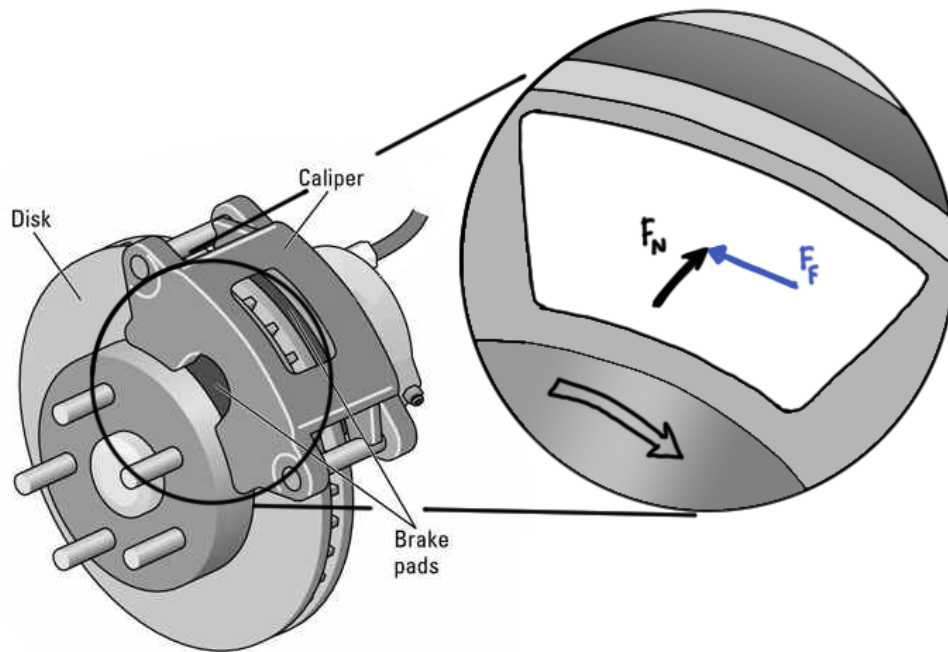
$$r_{min} \approx r_{max}$$

$$\text{Torque, } T = F_F \cdot r = \mu \cdot F_N \cdot r$$

Torque is limited by  $P_{max}$  & geometry

# A simple disc brake/clutch

The same principles apply when the geometry of the frictional surfaces change, as in the example of the disc brake/clutch



Normal force is distributed evenly

Friction force is distributed evenly

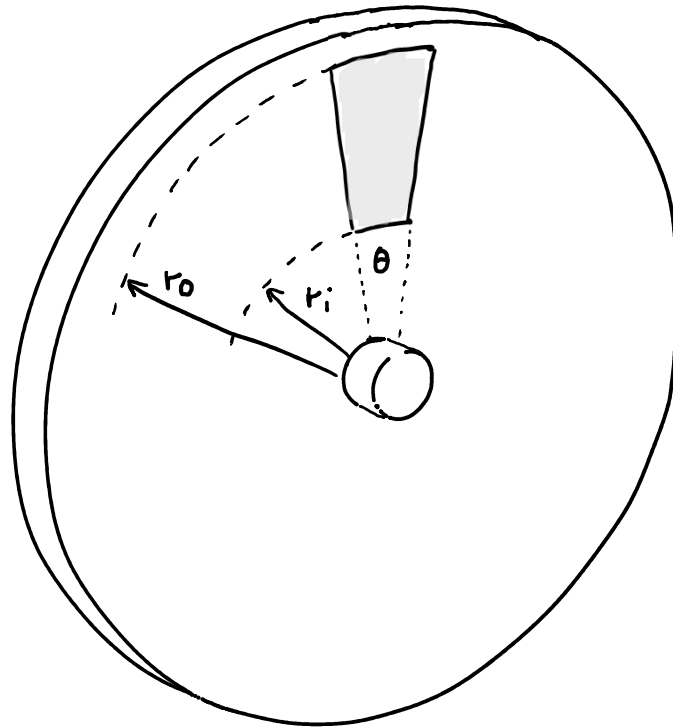
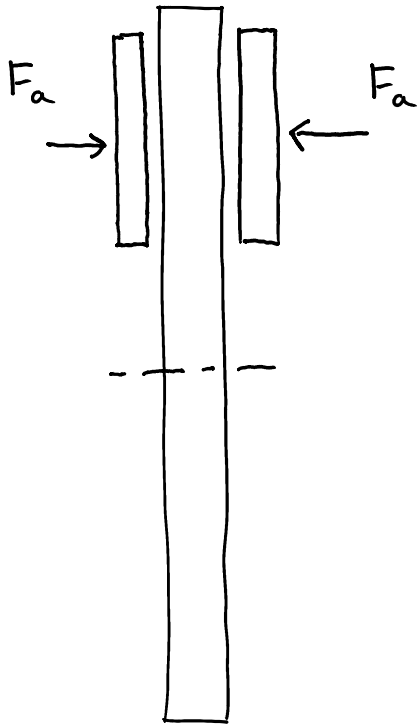
But radius is no longer large

$$r_{\min} \ll r_{\max}$$

$$\text{Torque, } T = \int F_F \cdot r \, dr = \int \mu F_N \cdot r \, dr$$

$F_N$  is still limited by  $P_{\max}$

# Disc brake simple pad geometry



$$\text{Area of contact} = \theta \cdot r_i \cdot (r_o - r_i)$$

$$F_a = P_{\max} \cdot N \cdot \text{Area}$$

$$\text{Area of Annulus sector} = \frac{\theta}{2} (r_o^2 - r_i^2)$$

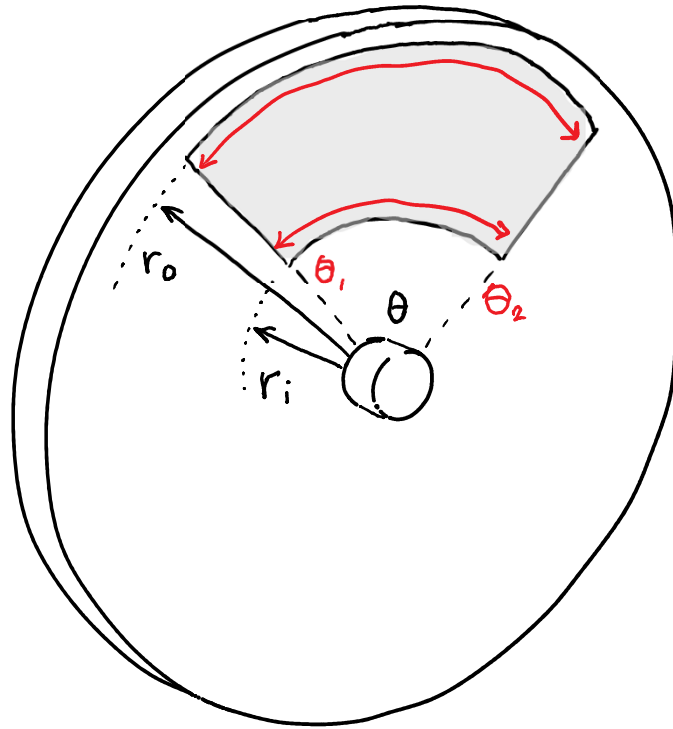
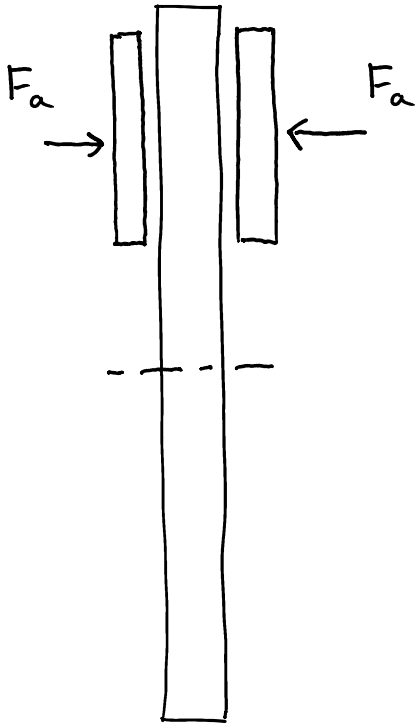
When  $\theta$  is small we only need to integrate over radius

$$T = \int_{r_i}^{r_o} N \cdot \mu \cdot F_a \cdot r \, dr$$

$$T = N \cdot \mu \cdot F_a \left( \frac{r_i + r_o}{2} \right)$$

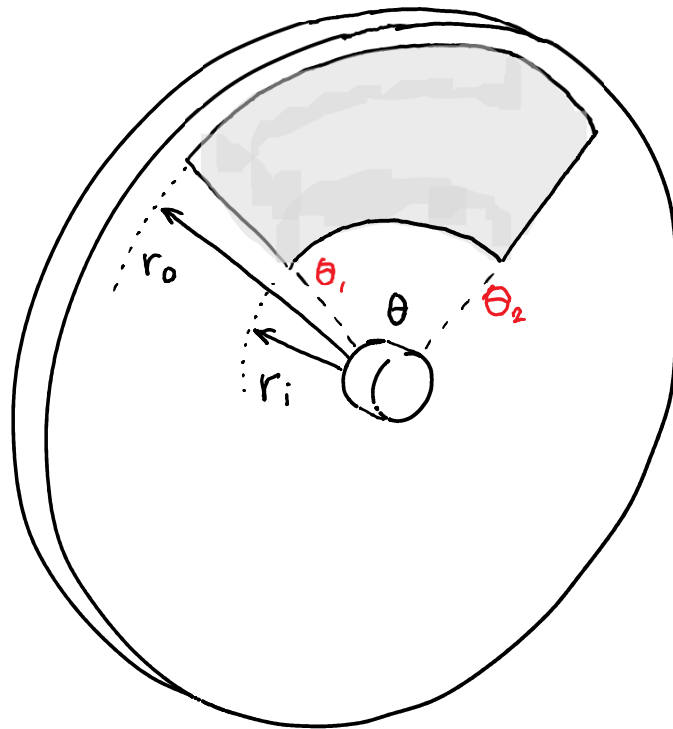
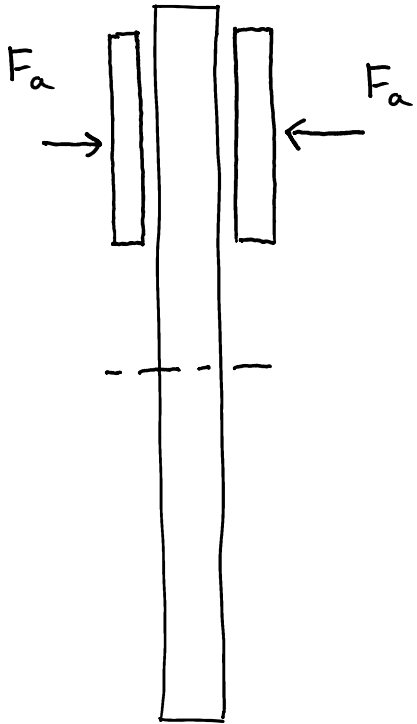
$N$  is the number of frictional contacts. In this case  $N = 2$

# Disc brake wide pad geometry



As  $\theta$  becomes large we have more area at a longer moment arm (as radius increases) and it becomes necessary to perform a double integration.

## Disc brake wide pad geometry continued



Assuming pressure is uniform

$$T = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} N \cdot \mu \cdot P \cdot r \, dr \, d\theta$$

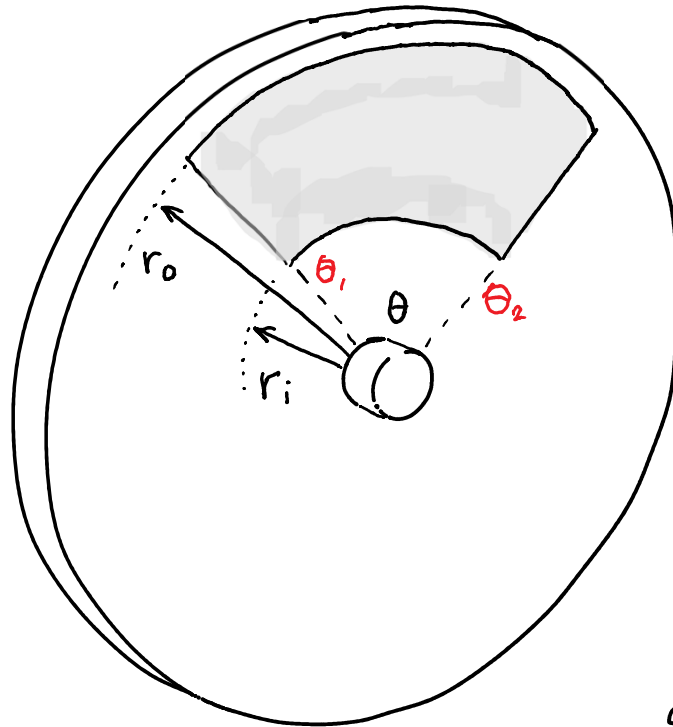
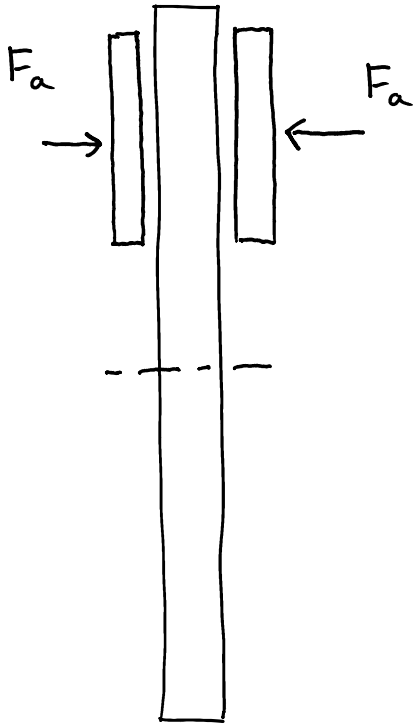
$$T = \frac{1}{3} (\theta_2 - \theta_1) \cdot N \cdot \mu \cdot P \cdot (r_o^3 - r_i^3)$$

$$F_a = \frac{1}{2} (\theta_2 - \theta_1) \cdot N \cdot P \cdot (r_o^2 - r_i^2)$$

This is likely to be the case when the brake is new and has no wear.



## Disc brake wide pad geometry continued

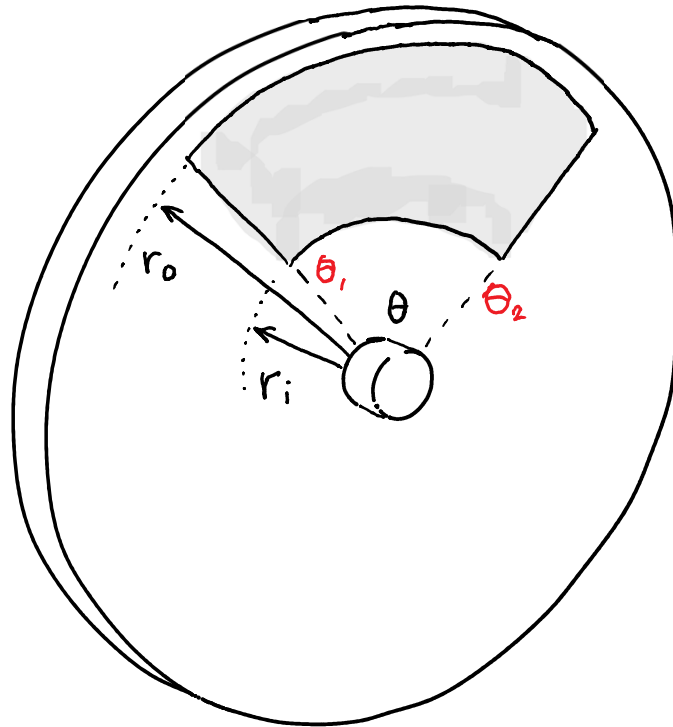
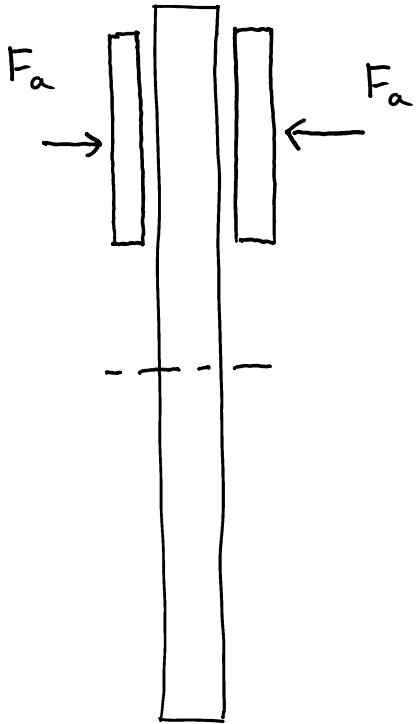


We know from our study of bearings that wear rate is proportional to contact force and sliding distance.

$$\text{Wear factor} = \frac{\text{volume [mm}^3\text{]}}{\text{force [N] \cdot distance [m]}}$$

◦◦ we would expect higher wear at the outer edge of pad and disc.

## Disc brake wide pad geometry continued



Assuming pressure varies with wear.

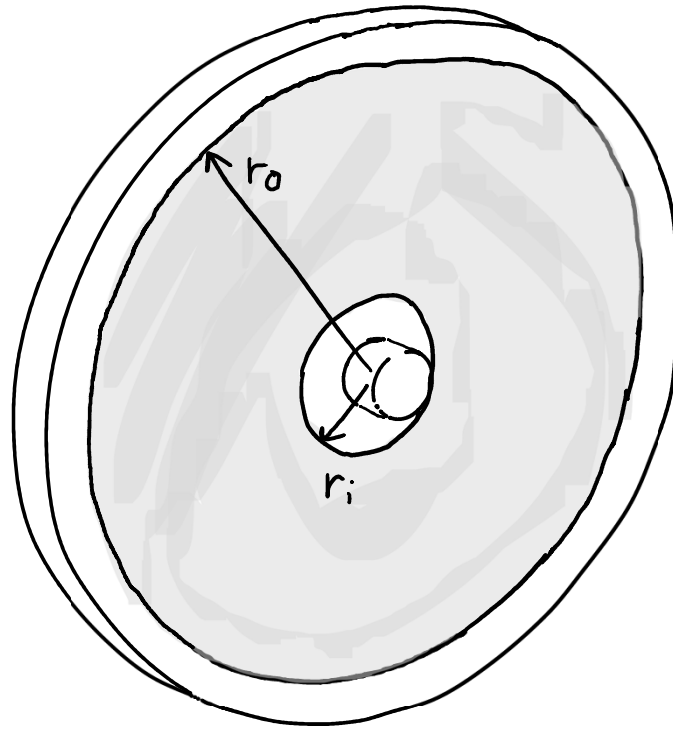
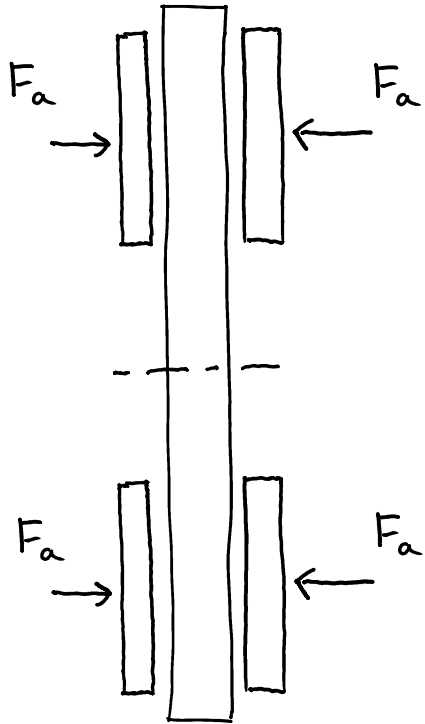
$P \cdot r$  becomes a constant  
and our integral becomes

$$T = \frac{1}{2} (\theta_2 - \theta_1) \cdot N \cdot \mu \cdot P \cdot r_i \cdot (r_o^2 - r_i^2)$$

$$F_a = (\theta_2 - \theta_1) \cdot N \cdot P \cdot r_i \cdot (r_o - r_i)$$

This is likely to be the performance when  
the brake or clutch is in normal operation.

Axial plate clutch In the special case  $\theta = 2\pi$



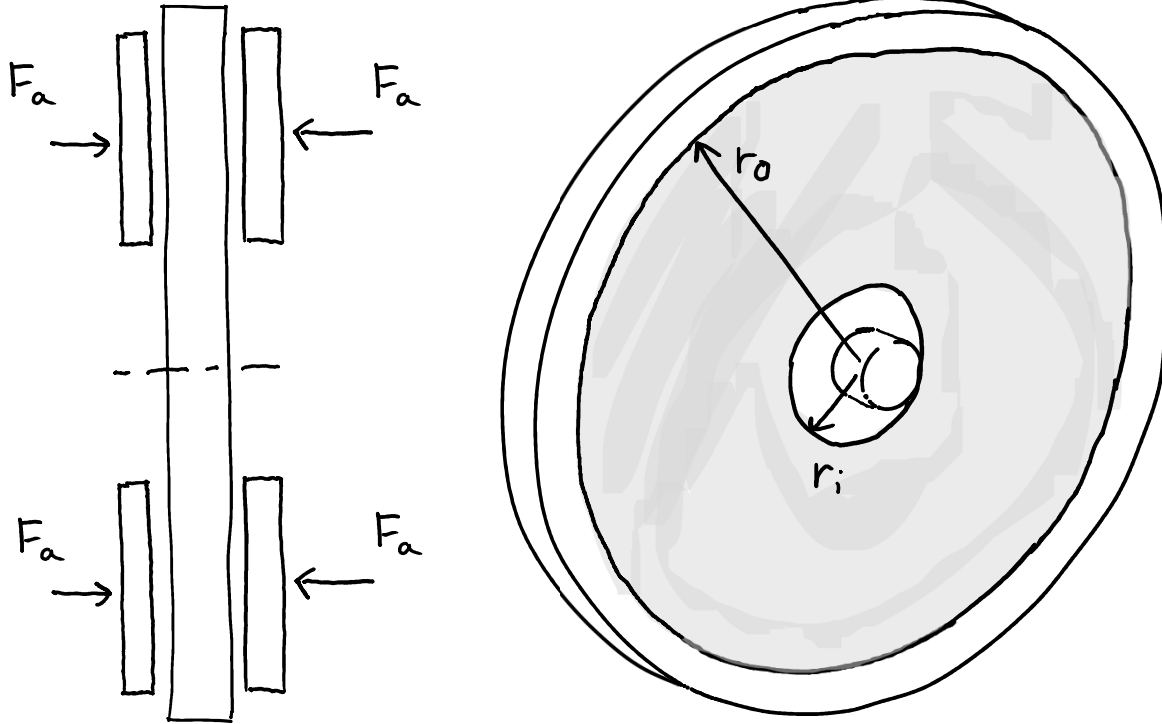
$$\text{Area} = N \cdot \pi (r_o^2 - r_i^2)$$

$$F_a = N \cdot P \cdot \pi (r_o^2 - r_i^2) \text{ for constant pressure scenarios.}$$

$$T = \int_{r_i}^{r_o} 2\pi \cdot \mu \cdot N \cdot P \cdot r^2 \, dr$$

$$T = \frac{2}{3} \pi \cdot \mu \cdot N \cdot P (r_o^3 - r_i^3)$$

# Axial plate clutch



In the worn condition where

$P \cdot r$  is a constant

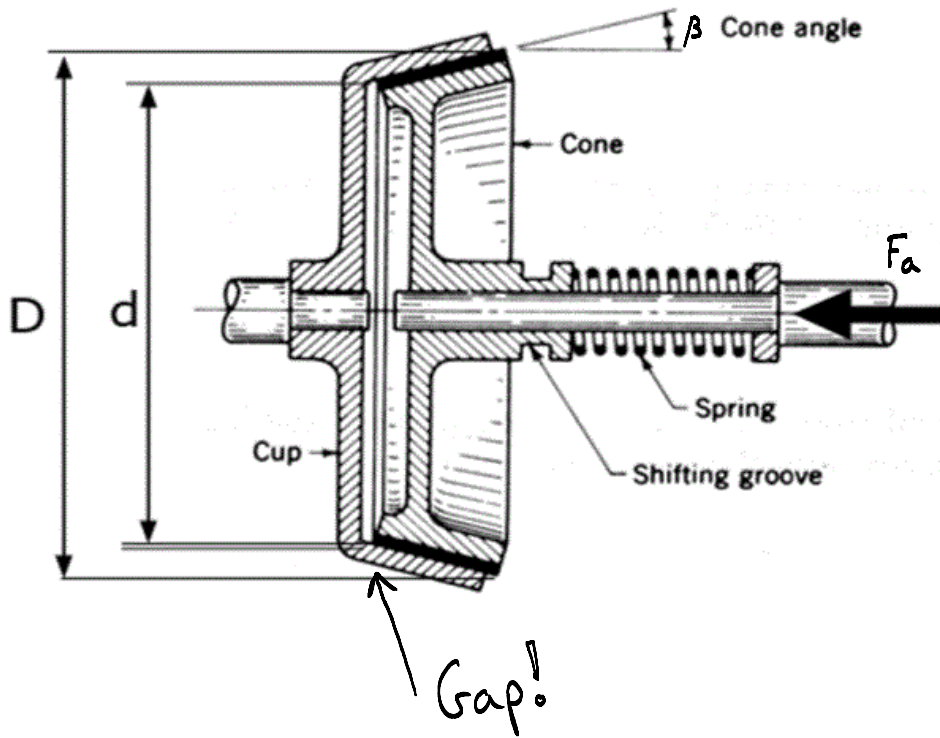
$$T = \frac{\pi \cdot \mu \cdot N \cdot P \cdot 2 \cdot r_i (2r_o^2 - 2r_i^2)}{8}$$

$$F_a = \pi \cdot N \cdot P \cdot r_i \cdot (2r_o - 2r_i)$$

This will be the most likely performance in long-term operation.

Designers must carefully consider which set of calculations best matches their application conditions.

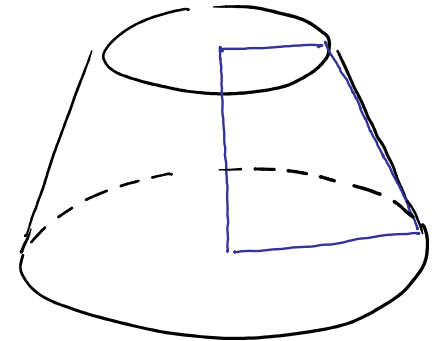
# Cone clutch

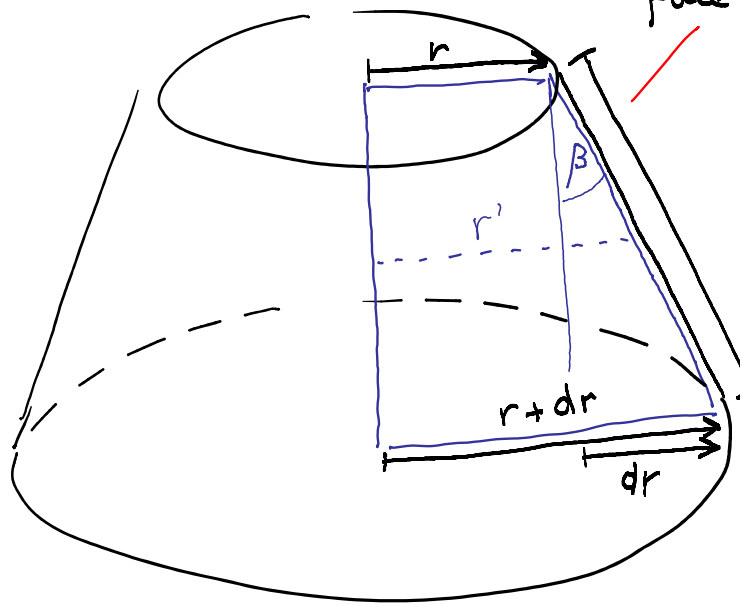


High torque transfer in a small volume.  
Cone acts like a wedge, multiplying the axial force for cone angles  $< 45^\circ$ .

Can have issues with release.  
Same principle, normal force over a frictional contact area.

Truncated cone  
or  
conical frustum.





Length of cone  
face =  $\frac{dr}{\sin \beta}$  [1]

Average radius  
 $r' = \frac{r + (r + dr)}{2}$

Circumference  
=  $2\pi r'$

For short cones  
Circumference  
=  $2\pi r$

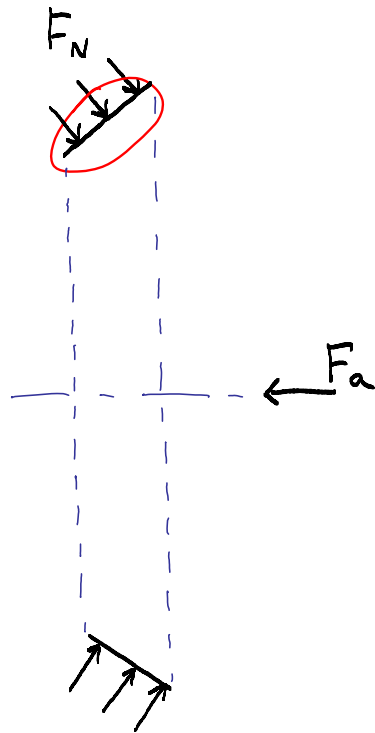
Area is face length times circumference.

$$\text{Area} = \frac{2\pi r \cdot dr}{\sin \beta} \quad [2]$$

When the cone clutch is new and there is no wear, we can assume that contact pressure is uniform and so is the normal force.

$$F_N = \frac{p \cdot 2\pi r \cdot dr}{\sin \beta} \quad [3]$$

Uniform pressure means uniform normal force



$$F_N = \frac{p \cdot 2\pi r \cdot dr}{\sin \beta} \quad [3]$$

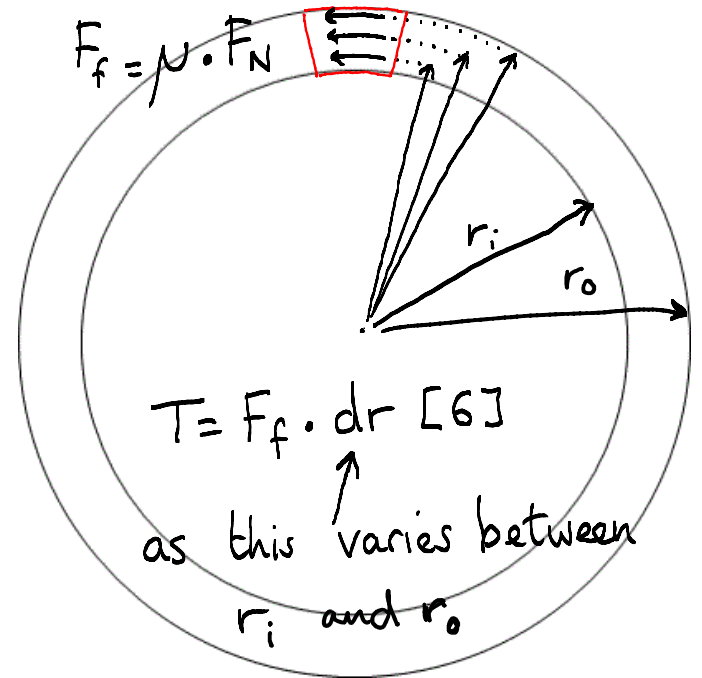
$$F_f = \frac{\mu \cdot p \cdot 2\pi r \cdot dr}{\sin \beta} \quad [5]$$

also  $F_a = F_N \cdot \sin \beta$

$$\therefore F_a = \frac{p \cdot 2\pi r \cdot dr \cdot \sin \beta}{\sin \beta}$$

$$F_a = p \cdot 2\pi r \cdot dr \quad [4]$$

Uniform normal force means that at any element the friction force is the same.



Expressing [4] and [6] as integrals across that range we get.

$$T = \frac{\mu \cdot 2\pi}{\sin\beta} \int_{r_i}^{r_o} p \cdot r^2 \cdot dr \quad [7]$$

$$F_a = 2\pi \int_{r_i}^{r_o} p \cdot r \cdot dr \quad [8]$$

We can now make one of two assumptions.

1) The clutch has perfect geometry and  $p$  is constant.

[8] becomes

$$F_a = \pi \cdot p (r_o^2 - r_i^2) \quad [9]$$

[7] becomes

$$T = \frac{\mu \cdot 2\pi \cdot p}{3 \cdot \sin\beta} (r_o^3 - r_i^3) \quad [10]$$

rearranging [9] for  $p$  we get

$$p = F_a / \pi \cdot (r_o^2 - r_i^2)$$

substituting into [10]

$$T = \frac{2\mu F_a (r_o^3 - r_i^3)}{3 \sin\beta (r_o^2 - r_i^2)} \quad [11]$$

$$F_a = \frac{3 \cdot \sin\beta (r_o^2 - r_i^2) \cdot T}{2 \cdot \mu \cdot (r_o^3 - r_i^3)} \quad [12]$$



This approach only applies when there is no wear.

2) Our alternative assumption is that wear has occurred. Because the outer edge of the cone has larger circumference, it slides further and wears faster.

The result is that  $(p \cdot r)$  is now constant.

i.e. the inner edge has higher pressure (from minimal wear) but a smaller radius. The outer edge has less pressure (due to wear) but a larger radius.

Now our integrals become:

$$F_a = 2\pi \cdot p \cdot r_i (r_o + r_i) \quad [13]$$

$$T = \frac{\mu \cdot \pi \cdot p \cdot r_i (r_o^2 - r_i^2)}{\sin \beta} \quad [14]$$

rearrange

$$p = \frac{F_a}{2\pi r_i (r_o + r_i)} \quad [15]$$

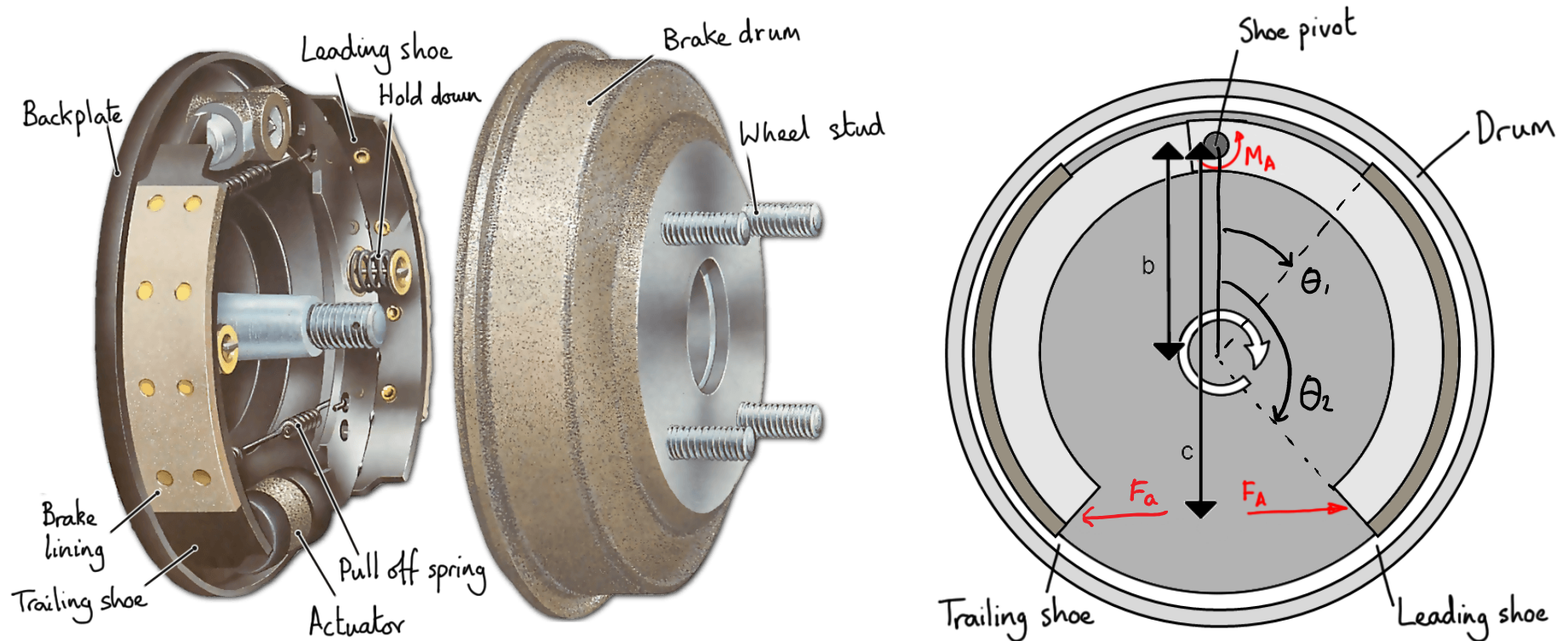
Substitute into [14]

$$T = \frac{\mu F_a \cdot (r_o + r_i)}{2 \sin \beta} \quad [16]$$

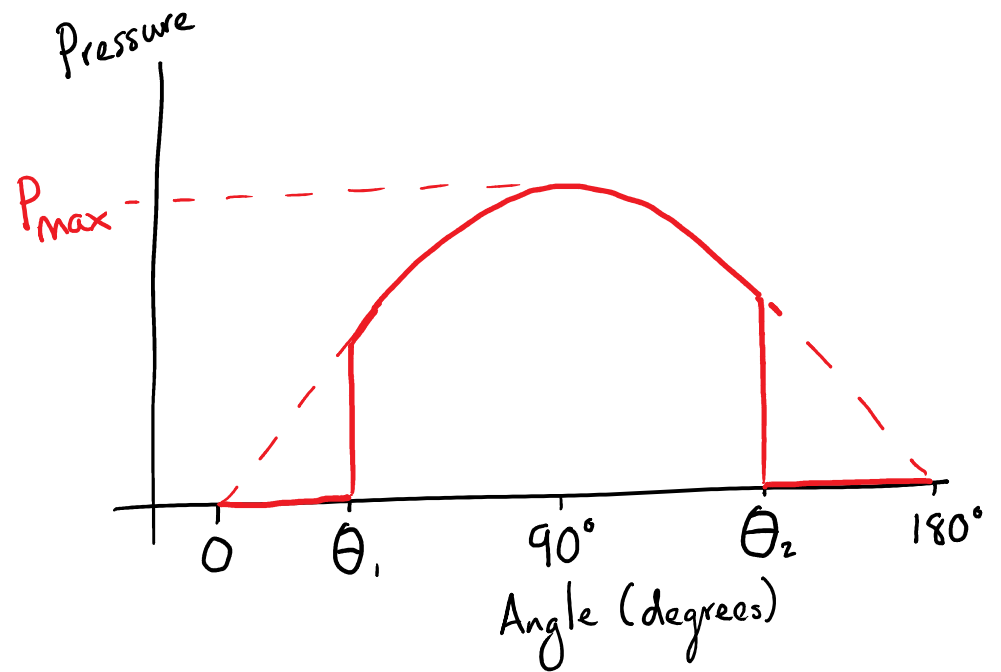
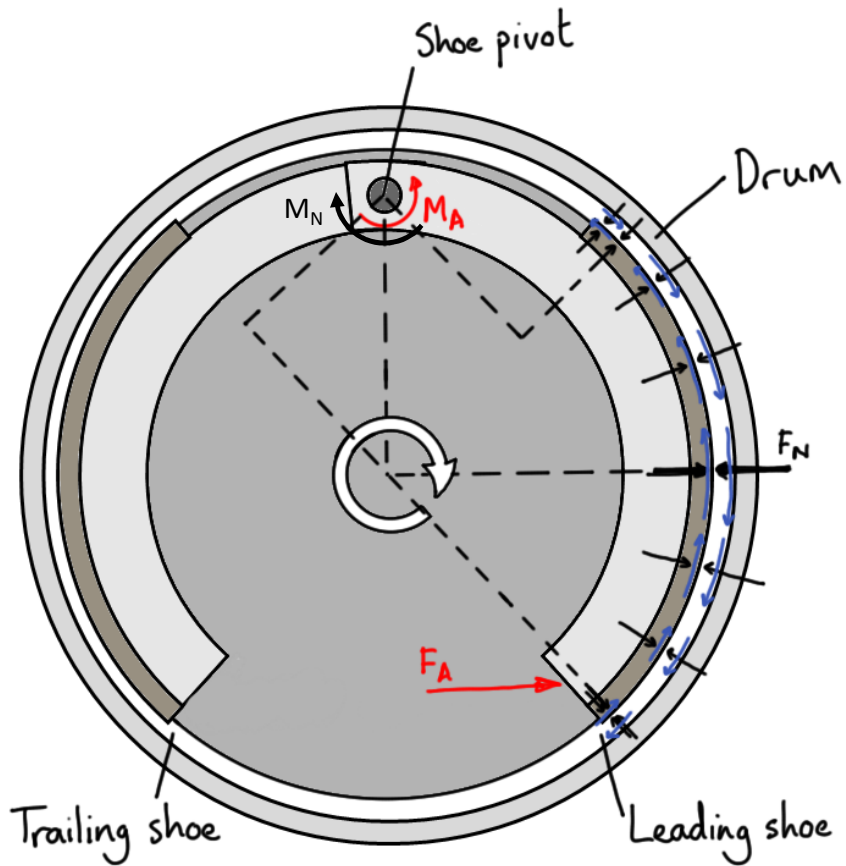
$$F_a = \frac{2 \sin \beta}{\mu (r_o + r_i)} \cdot T \quad [17]$$

# Key principles – Drum brakes/clutches

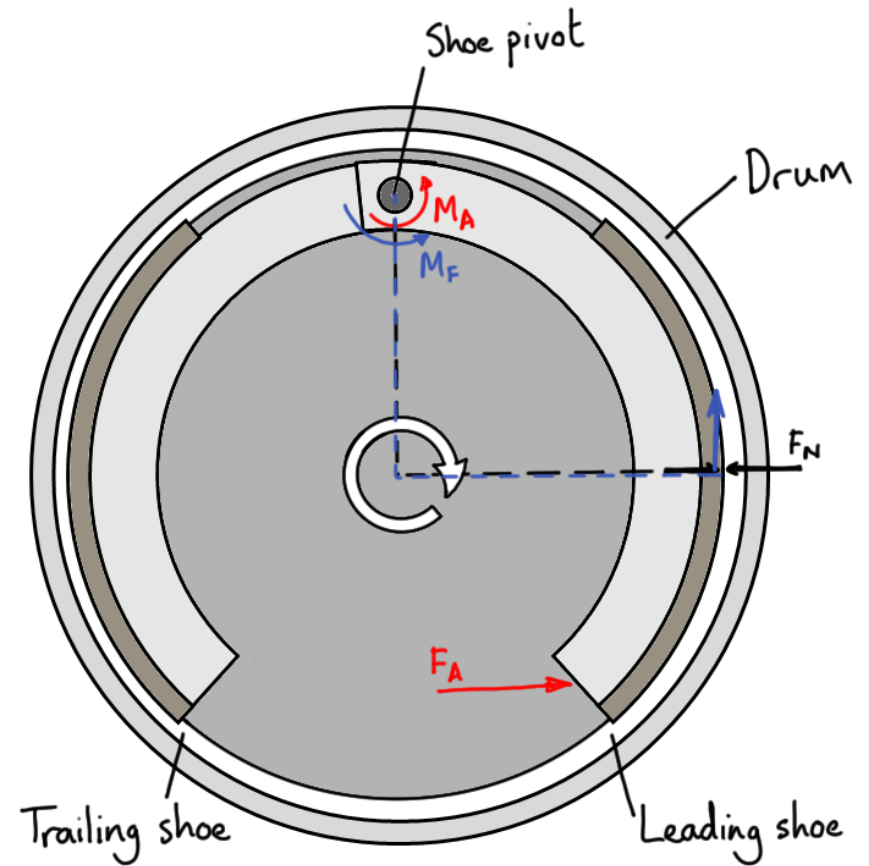
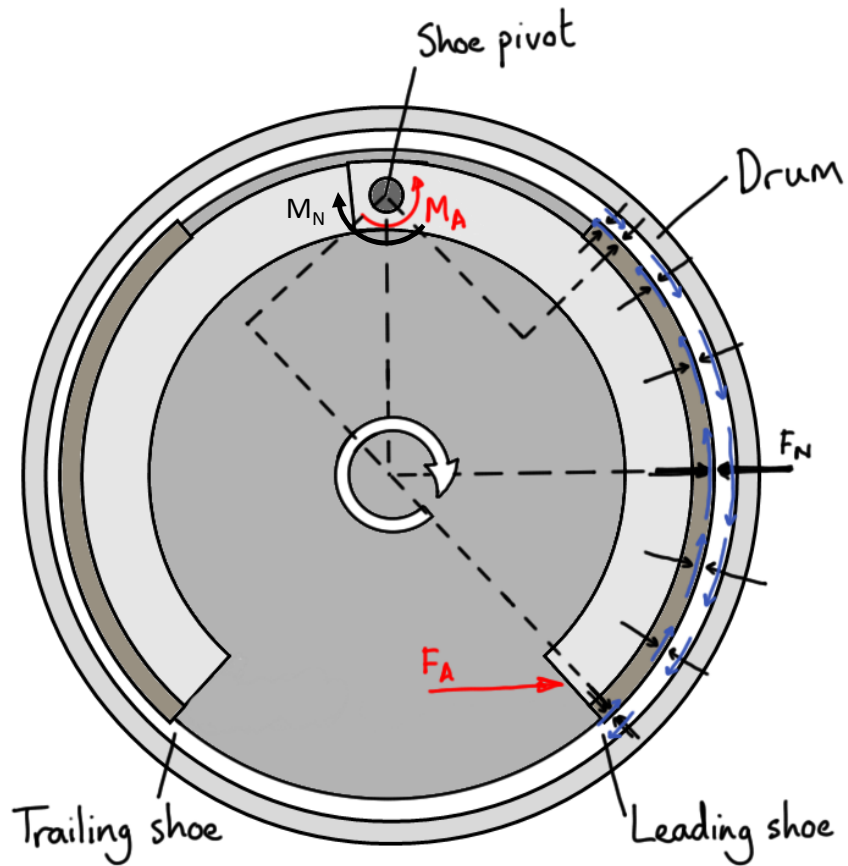
For drum geometries the frictional contact surface is at constant radius but varying normal force



# Key principles – Drum brakes/clutches



# Key principles – Drum brakes/clutches



## Leading shoe: braking torque capacity

Determine braking torque applied by leading shoe

$$T_L = \int r \times \mu dF_n = \mu w r^2 \frac{p_{\max}}{\sin \theta_{\max}} (\cos \theta_1 - \cos \theta_2)$$

normal forces

max pressure felt by lining  
(must be  $\leq$  to suppliers spec.)

drum radius

angle at which max pressure occurs

w = width

b = the distance from the pivot to the centre of rotation

r = the radius at which the shoe contacts the drum, with respect to the centre of rotation

c = the distance from the pivot to the brake actuators

## Leading shoe: normal moment

Determine the normal moment from the leading shoe

$$M_n = \int l_{\perp pivot} \cdot dF_n = \frac{wrbp_{\max}}{\sin \theta_{\max}} \left[ \frac{1}{2} (\theta_2 - \theta_1) - \frac{1}{4} (\sin 2\theta_2 - \sin 2\theta_1) \right]$$

normal forces

perp. distance to pivot

w = width

b = the distance from the pivot to the centre of rotation

r = the radius at which the shoe contacts the drum, with respect to the centre of rotation

c = the distance from the pivot to the brake actuators

## Leading shoe: frictional moment

Determine the frictional moment from the leading shoe

$$M_f = \int l_{\perp \text{ pivot}} \cdot \mu dF_n = \frac{\mu w r p_{\max}}{(\sin \theta)_{\max}} \left[ r(\cos \theta_1 - \cos \theta_2) + \frac{b}{4} (\cos 2\theta_2 - \cos 2\theta_1) \right]$$

normal forces

perp. distance to pivot

w = width

b = the distance from the pivot to the centre of rotation

r = the radius at which the shoe contacts the drum, with respect to the centre of rotation

c = the distance from the pivot to the brake actuators

## Leading shoe - Sum moments to zero (equilibrium)

$$0 = cF_a + M_f - M_n$$

Sum moments about the shoe pivot

additive for leading shoe

"Self Locking"

$$\text{if } M_n \leq M_f$$

$$\text{then } F_a \leq 0$$

lining will "grab" and lock to drum on contact

$$F_a = \frac{M_n - M_f}{c}$$

w = width

b = the distance from the pivot to the centre of rotation

r = the radius at which the shoe contacts the drum, with respect to the centre of rotation

c = the distance from the pivot to the brake actuators



## Trailing shoe - Sum moments to zero (equilibrium)

$$0 = cF_a - M_f - M_n$$

Sum moments about the shoe pivot

Subtractive for trailing shoe

$$F_a = \frac{M_n + M_f}{c}$$

Trailing shoes are always de-energising

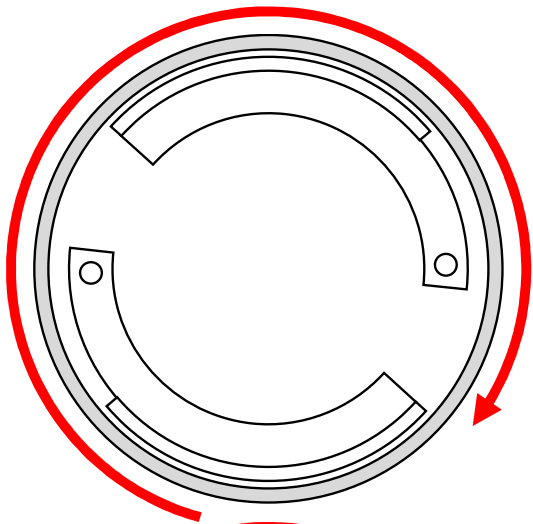
If your design has both leading and trailing shoes you will normally design for the leading shoe first as the self-energising effect will mean this shoe always has the higher maximum pressure and will exceed material pressure limit at lower actuation force

w = width

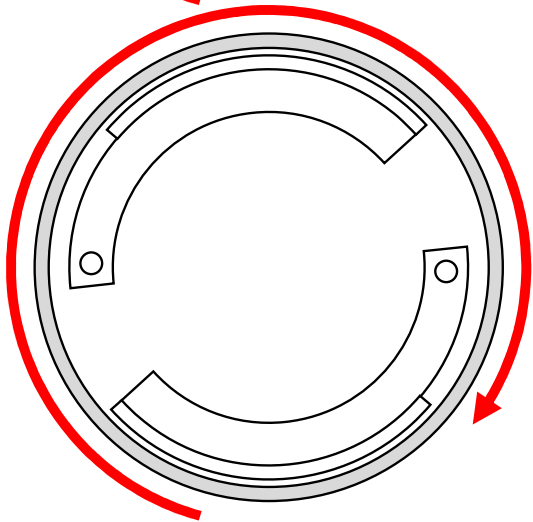
b = the distance from the pivot to the centre of rotation

r = the radius at which the shoe contacts the drum, with respect to the centre of rotation

c = the distance from the pivot to the brake actuators

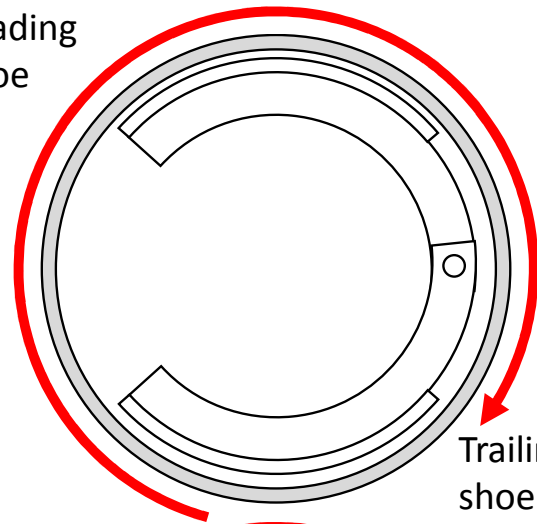


**Leading – Leading brake**  
 High breaking force  
 Brakes are self engaging  
 Will remain engaged even  
 if the actuation force is  
 removed.



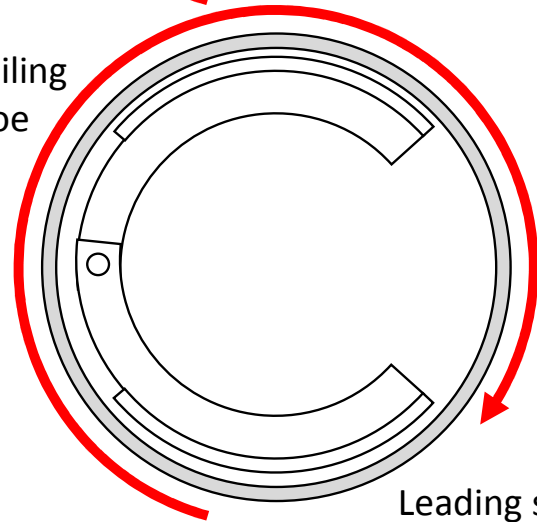
**Trailing - Trailing brake**  
 Low breaking force  
 Brakes self releasing  
 Actuation force must be  
 maintained to maintain  
 brake torque.

Leading  
shoe



Trailing  
shoe

Trailing  
shoe



Leading shoe

# Summary

- Brakes and clutches both use friction to bring rotating systems to the same velocity
- The frictional materials for contacts must:
  - Have high and uniform coefficient of friction
  - Be unaffected by moisture and heat
  - Have good heat conductivity
  - Withstand high pressures
  - Withstand wear and scoring
- Torque is limited by  $P_{\max}$  and the contact area
- Disc systems have varying radius and nominally constant pressure
- Drum systems have nominally constant radius but varying pressure
- Wear can cause the performance of the systems to degrade. Performance loss is greater for disc/plate/cone systems due to greater wear at larger radius
- Long shoe systems must consider leading/trailing orientation self-energise/de-energise

